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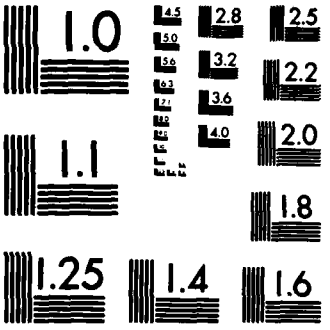

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Research Report CCS 457

HOMOMOLLIFICATION PRESERVES REAL ESSENTIALITY

by

A. Charnes
B. Golany

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ABSTRACT

We show that the homomollifier w is inessential iff all (n-1)-person subgames of the original game v are inessential. Further, v is "really" essential iff all (n-1)-person subgames of homomollifier w are essential. Thus, homomollification preserves (and maximally increases) real essentiality.

KEY WORDS

Homomollifier

Real Essential n-person Games

Characteristic Function Games

HOMOMOLLIFICATION PRESERVES REAL ESSENTIALITY

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In [1], Charnes, Rousseau, and Seiford defined the homomollifier w of an essential superadditive n-person characteristic function game v as:

$$(1) \qquad w(S) \triangleq \frac{|S|}{n}\,\overline{v}(S) + \frac{n - |S|}{n}\,v(S) \;,\; S \subseteq N$$

where $\overline{v}(S) \triangleq v(N) - v(N-S)$.

They showed that w is superadditive and constant sum among other convenient properties. They did not consider, however, whether or not essentiality is preserved under homomollification. It is the purpose of this note to evaluate this situation via "real" essentiality, i.e., some (n-1)-person subgames of v is essential.

We show that the homomollifier w is inessential iff all (n-1)-person subgames of the original game v are inessential. Further, v is "really" essential iff all (n-1)-person subgames of homomollifier w are essential. Thus, homomollification preserves (and maximally increases) real essentiality.

Theorem: The homomollifier w of an essential superadditive game v is inessential iff all (n-1)-person subgames of v are inessential.

Proof: Let w be the homomollifier of v. Then

$$(2) \qquad w(i) = \frac{1}{n}\,\overline{v}(i) + (1 - \frac{1}{n})v(i)$$

$$= v(i) + \frac{1}{n}[v(N) - v(N-\{i\}) - v(i)]$$

If the subgame of v without player i is essential, then

$$(3) \qquad w(i) < v(i) + \frac{1}{n}[v(N) - \sum_{j \neq i} v(j) - v(i)]$$

$$= v(i) + \frac{1}{n}[v(N) - \sum_{k} v(k)]$$

So,

$$\sum_{j} w(j) < \sum_{j} v(j) + v(N) - \sum_{k} v(k) = v(N) \quad .$$

If w is inessential, $\sum_{j} w(j) = w(N) = v(N)$, so that

$$(4) \qquad v(N) = \sum_{j} w(j) < v(N) \text{ , a contradiction.}$$

Hence w inessential implies all (n-1)-person subgames of v are inessential.

Conversely, if all (n-1)-person subgames are inessential, then

$$(5) \qquad w(i) = v(i) + \frac{1}{n}[v(N) - \sum_{k} v(k)] \text{ ,}$$

from the preceding.

So,

$$\sum_{i} w(i) = \sum_{i} v(i) + v(N) - \sum_{k} v(k) = v(N) \text{ .}$$

But $v(N) = w(N)$, so w is inessential.

Q.E.D.

Corollary: The homomollifier w of a really essential superadditive game v is maximally really essential.

Proof: By definition, v is really essential iff some (n-1)-person subgame of (N,v) is essential.

By the Theorem, the homomollifier w must be essential if v is really essential.

Suppose that the (n-1)-person subgame of w which omits player i is inessential.

Then

$$(6) \qquad w(N - \{i\}) = \sum_{j \neq i} w(j) \ .$$

But by the constant sum property of w,

$$(7) \qquad w(N) = w(i) + w(N - \{i\}) = w(i) + \sum_{j \neq i} w(j)$$

so that w is inessential, a contradiction.

Thus <u>every</u> (n-1)-person subgame of w must be essential, i.e., the homomollifier is maximally really essential.

Q.E.D.

The results above reinforce the considerations of Charnes and Golany [2] in defining a unique core-like solution concept, the homocore, in terms of the (n-1)-person and one person subgame levels. Dominance stability as reflected in essentiality or inessentially is hereby revealed to be resident in the properties of these levels.

REFERENCES

[1] A. Charnes, J. Rousseau, and L. Seiford, "Complements, Mollifiers and the Propensity to Disrupt," International Journal of Game Theory, Vol. 7, 1978, 37-50.

[2] A. Charnes and B. Golany, "Homocores, Cores and Operational Inefficiency in Superadditive N-Person Games," Center for Cybernetic Studies Research Report CCS 456, February 1983, The University of Texas, Austin, TX, 78712.

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We show that the homomollifier w is inessential iff all (n-1)-person subgames of the original game v are inessential. Further, v is "really" essential iff all (n-1)-person subgames of homomollifier w are essential. Thus, homomollification preserves (and maximally increases) real essentiality.

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